First-Order differential equation

Solve the following differential equation:

$$2(x + y\,y') + e^{y}(1 + x\,y') = 0$$

Solution

Reorganizing:

$$2x + 2yy' + e^y + xe^y y' = 0$$

$$2y\frac{dy}{dx} + xe^y\frac{dy}{dx} = -2x - e^y$$

$$(2y + xe^y)dy + (2x + e^y)dx = 0$$

We compute the derivatives:

$$e^y$$

Since they are equal, we now calculate the integrals to solve this exact differential equation:

$$\int (2y + xe^y)\, dy = y^2 + xe^y + B$$

$$\int (2x + e^y)\, dx = x^2 + xe^y + D$$

Finally, the solution is:

$$y^2 + x^2 + xe^y = C$$